Using Belief Functions in Software Agents to Test the Strength of Application Controls: A Conceptual Framework

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ABSTRACT

Belief functions have been used to model audit decision making for over 20 years. More recently they have been used in assessing the strength of internal controls and information systems security. There has been some research on software agents in auditing, particularly in the web search bot area [Nelson *et al.*, 2000]. This research extends the work of Srivastava and others [Bovee *et al.*, 2007; Srivastava and Shafer, 1992; Srivastava, 1997] in belief functions and Nehmer [Nehmer, 2003, 2009] in the use of software agents in internal control evaluations. It looks at the problem of assuring the adequacy of application internal controls in highly automated transaction processing environments.

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INTRODUCTION

Belief functions have been used to model audit decision making for over 20 years. More recently they have been used in assessing the strength of internal controls and information systems security. There has been some research on software agents in auditing, particularly in the web search bot area [Nelson *et al.*, 2000]. This research extends the work of Srivastava and others [Bovee *et al.*, 2007; Srivastava and Shafer, 1992; Srivastava, 1997] in belief functions and Nehmer [Nehmer, 2003, 2009] in the use of software agents in internal control evaluations.

This paper looks at the problem of assuring the adequacy of application internal controls in highly automated transaction processing environments. The research focuses on risk management, systems of internal controls, and transaction processing environments. In this setting, investments in systems of internal controls are justified by their risk reducing properties. By extending the framework reported in [Nehmer, 2009] into an application setting, the domain structure is defined in a way to allow the implementation of systems of internal controls as systems of agents which perform control monitoring activities.

There has been a lot of theoretical work done on building stable agent communities. [Holland, 1995] is a very assessable first pass at some of this work. [Fingar, 1998] and [Farhoodi, 1997] discuss agent systems from an executive, decision making perspective. There have been few formal attempts to define systems of internal controls in the accounting literature. The system defined in this project is based on the risk reducing monitoring activities of a community of software agents.

The research constructs a conceptual model which uses belief functions to determine whether there is sufficient evidence to support the decision to rely on a set of automated application controls. The model defines sufficiency in terms of the closure properties of the belief function. Although interest in using software agents for control of ecommerce applications and continuous auditing has surfaced in recent years, there has not been much formal work on how to apply agent technologies in financial control environments. This research moves this important area forward by providing a conceptual model of the agent community within an automated transaction processing environment.

PRIOR LITERATURE

Belief functions have been used in auditing and the internal control literature for a number of years. The approach we rely on derives from [Mock *et al.*, 2009]. That paper introduces a belief function approach to risk assessments and internal control systems. It does this in the context of the post Sarbanes Oxley era. The paper gives a method for combining evidence from multiple sources. The method provides a "rigorous algorithm to …, propagate and aggregate the results, and output quantitative risk assessments" (p. 66). We apply their methodology on the individual control procedure level for aggregations of control procedures. We use this method in the model building section of this paper. Our approach also relies on [Srivastava, 2005] which addresses the belief function formulation for binary variables.

There has been comparatively little work done with intelligent agents in accounting. Of that, most references are to the search bots for intelligence gathering and classifying in internet web page searches. In the accounting domain, the natural search target is financial data and the natural location of the data is the web pages of the Securities and Exchange Commission and its EDGAR system. [Nelson *et al.*, 2000] provide a good overview of the research potential in this area. The paper suggests that agents “do the more mundane and manual tasks of the audit function” (p. 242). The authors suggest that the internet provides the opportunity for audit firms to provide their clients with new services. They classify these services into two types: quality and service. The paper breaks each of the two types into five sub-types. The quality dimension breaks down transactions and data into verification, authentication, integrity, completeness and timeliness. The service types break down into Nonrepudiation, proxy intelligence searches, real time database search and reporting, natural language translation, and competitive intelligence. The agent system modeled in our research focuses on two sub-types of quality and one sub-type of service. The two quality sub-types are integrity and completeness. The service sub-type is competitive intelligence which [Nelson *et al.*, 2000] define as “valuable and salable client competitive knowledge” (p. 243). Our agents use beliefs about the operation of the system of internal control to help firms determine when the system is losing effectiveness and compromising integrity and/or completeness. This information is useful for maintaining the firm’s competitive position.

[Nehmer, 2003] models a generalized community of transaction agents within e-commerce environments. The software agents in that paper are characterized by autonomy, flexibility and threads of control. Agent activities are mapped into the COSO and COBIT internal control frameworks. This allows a grounding of the design of the agent community in risk assessment and continuous monitoring. Our current research adds detail to this model by considering a specific case in a sales ordering and fulfillment business context. [Nehmer, 2009] provides a description and simulation of agent communities but where the agents are constructed to represent firms rather than controls. Our research builds agent communities of control procedures.

In the next section we develop the business situation in which we model the agent community. After that we develop the belief function model followed by some computational examples. We then discuss the results and their implications. The final section discusses possible future lines of research in this area.

THE BUSINESS CASE AND BELIEF-FUNCTION MODEL

The situation that we model in this paper involves a credit sales and delivery process. The business context has been considerably simplified for clarity of discussion. However, the extensions into a real-world business context are straight forward, involving only scale up. The characterization of the agents is as follows.

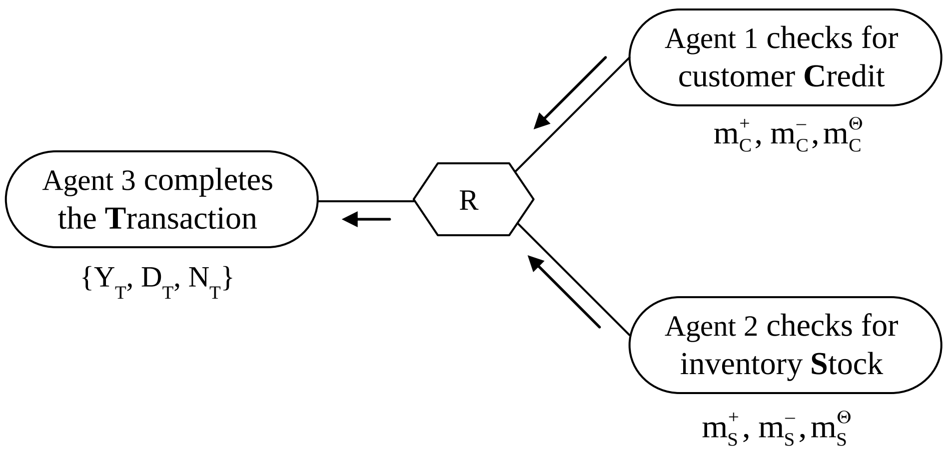
Agents:

1. Reside in a web based store front.
2. The company has two policies for its sales and delivery process. The policies require that: 1) all credit sales are verified by the credit card company and an approval code is added to the transaction before it is sent for processing and, 2) customers are advised if items are not in stock with a popup message during the ordering process. Execution of this message is also tagged to the transaction indicating when items are out of stock.
3. Agents reside in the middle ware layer, receiving transactions from the store front, performing some translation and pre-processing and sending the transactions on to the back end shipment and accounting systems.
4. Agents check for the approval code on all credit sales and for the out of stock code on all sales.

In the next section we develop a belief function model of the above business case. Such a model can be used to show change in belief as additional evidence is collected during the period or as a report of overall belief. The report is generated at the end of the evidence collection period which usually corresponds to the end of the audit period. During the period the agents can be queried as to the current belief state.

A Belief –Function Model

The following evidential diagram represents a belief-function model of the business process described above. This model consists of three agents represented by oval shaped boxes. The three agents are related through a relationship R, which is described later in the section.



The following discussion provides a detailed description of the three agents along with the interrelationship.

1. Agent 1 checks for customer credit: There are two possible states $\{Y_C, N_C\}$:

    Y_C = Yes, the credit is approved

    N_C = No, the credit is not approved

    The belief masses pertaining to the above states are represented by the following symbols:

$m(\{Y_C\}) = m_C^+$, $m(\{N_C\}) = m_C^-$, $m(\{Y_C, N_C\}) = m_C^\Theta$

2. Agent 2 checks whether inventory stock is available: There are two possible states $\{Y_S, N_S\}$:

   Y_S = Yes, the item is available

   N_S = No, the item is not available

   The belief masses pertaining to the above states are represented by the following symbols:

   $m(\{Y_S\}) = m_S^+$, $m(\{N_S\}) = m_S^-$, $m(\{Y_S, N_S\}) = m_S^\Theta$

3. Agent 3 completes the transaction, i.e., the agent sends the signal to shipping department for shipment of the item. There are three possible states $\{Y_T, D_T, N_T\}$:

   Y_T = Yes, Complete the transaction, i.e., ship the item

   D_T = Delay the shipment, i.e., inform the customer that item will be shipped at a later date (item is on the back order)

   N_T = No, do not ship the item

Agent 3 acts upon the information received from Agent 1 and Agent 2. The following actions of Agent 3 are defined based on the various possible states of Agent 1 and Agent 2.

$Y_T = Y_CY_S$ (Yes complete the transaction, i.e., ship the item since credit is approved and Yes, the item is available)

$D_T = Y_CN_S$ (Delay the shipment of item, credit is approved but the item is not in the stock)

$N_T = \{N_CY_S, N_CN_S\}$ (Do not complete the transaction, i.e., do not ship the item because the credit is not approved whether the item is available or not)

The above condition is defined by the following belief mass for the interrelationship among the three agents:

$m_R(\{Y_CY_SY_T, Y_CN_SD_T, N_CY_SN_T, N_CN_SN_T\}) = 1$

In order to determine the beliefs and plausibilities for the action of Agent 3, we need to propagate belief masses from the two agents, Agent 1 and Agent 2. This process is achieved through the following steps.

Step 1: Vacuously extend the belief masses at Agent 1 to the joint space of the possible relationship, i.e., onto the space $\Theta = \{Y_CY_SY_T, Y_CN_SD_T, N_CY_SN_T, N_CN_SN_T\}$.

Step 2: Vacuously extend the belief masses at Agent 2 to the joint space of the possible relationship, i.e., onto the space $\Theta = \{Y_CY_SY_T, Y_CN_SD_T, N_CY_SN_T, N_CN_SN_T\}$.

Step 3: Combine the above belief masses obtained in Step 1 and Step 2 using Dempster's rule of combination and marginalize them onto the space $\{Y_T, D_T, N_T\}$. This step yields the following belief masses:

$m(\{Y_T\}) = m_C^+ m_S^+$

$m(\{D_T\}) = m_C^+ m_S^-$

$m(\{N_T\}) = m_C^-$

$m(\{Y_T, D_T\}) = m_C^+ m_S^\Theta$

$m(\{Y_T, N_T\}) = m_C^\Theta m_S^+$

$m(\{\{D_T, N_T\}) = m_C^\Theta m_S^-$

$m(\{\Theta\}) = m_C^\Theta m_S^\Theta$

These belief masses yield the following beliefs and plausibilities:

$\mathrm{Bel}(\{Y_T\}) = m_C^+ m_S^+ = \mathrm{Bel}(\{Y_C\}).\mathrm{Bel}(\{Y_S\})$, i.e., the belief that the item is shipped is equal to the product of the two beliefs that credit is approved and the item is available

$Bel(\{D_T\}) = m_C^+ m_S^- = Bel(\{Y_C\}).Bel(\{N_S\})$, i.e., the belief that the shipment is delayed is the product of the two beliefs that credit is approved and the items is not available

$Bel(\{N_T\}) = m_C^- = Bel(\{N_C\})$, i.e., the belief that shipment will not be made is equal to the belief that credit is not approved irrespective of whether the item is in the stock or not.

$Pl(\{Y_T\}) = Pl(\{Y_C\})Pl(\{Y_S\})$, i.e., the plausibility that the item is shipped is equal to the product of the plausibilities that credit is approved and the item is available.

$Pl(\{D_T\}) = Pl(\{Y_C\})Pl(\{N_S\})$, i.e., the plausibility that the shipment is delayed is the product of the plausibilities that credit is approved and the item is not in the stock.

$Pl(\{N_T\}) = Pl(\{N_C\})$, i.e., the plausibility that shipment is not made is equal to the plausibility that credit is not approved irrespective of whether the item is available or not.

$Pl(\{D_T\ N_T\}) = 1 - m_C^+ m_S^+ = 1 - Bel(Y_T)$.

One can perform sensitivity analysis as to the performance of Agent 3 based on the reliability of Agent 1 and Agent 2.

COMPUTATIONAL EXAMPLES

Next we perform some computational examples with the modeled specified in the previous section. In the first example, agent 1's belief that credit is approved varies from 0 to 1 while its belief that credit is not approved is held constant at 0. Similarly, agent 2's belief that the item is in inventory varies from 0 to 1 while its belief that the item is not in inventory is held constant at 0 (see Table 1). The effects of these manipulations on agent 3 are shown in Table 2. Table 2 contains the derived beliefs and plausibilities. Note that holding the negative belief (credit not approved, inventory not in stock) to 0 keeps the belief that credit is approved but the item not in stock at 0 as well. It also holds the plausibility that credit is approved and the item is available at 1. Figure 1 is a graphic representation of this situation showing that as the reliability of agents 1 and 2 increases from 0 to 1, belief that shipment will be made rises and plausibility that either shipment will be made or delayed falls.

The next example, agent 3 believes that agent 1 misreports bad credit as good credit. This situation is shown numerically in Table 3 and Table 4 and graphically in Figure 2. Note that we show agent 3's mistrust of agent 1 as agent 1's belief that bad credit is being reported, m_C^- . We then scale the results on the X-axis of Figure 2 to these, opposed, beliefs. The figure shows that agent 3's belief falls to 0 as agents 1's presumed reliability increases. This indicates that agent 3 is discounting agent 1's reported belief by using the compliment of agent 1's believes instead of agent 1's reported belief.

An extension of the last example occurs in cases where we consider that agent 3 can be designed to check on the reliability of agent 1 and 2's signals. Here we add two components to the belief function model: agent 3's scoring of both agent 1 and agent 2's historical reliability. The idea here is to add a scorecard function to the system which allows agent 3 to record the other agent's responses and the correct state of affairs determined at some *ex post* date. Agent 3 uses these scores to provide itself with a simple percentage reliability rating on the other two agents. We then extend the belief formulations as follows:

$\mathrm{Bel}(\{Y_T\}) = m_c^+ \, r_c \, m_s^+ \, r_s$

$\mathrm{Bel}(\{D_T\}) = m_c^+ \, r_c \; m_s^- \, r_s$

$\mathrm{Bel}(\{N_T\}) = m_c^- \, r_c$

where $Pl(\{D_T N_T\}) = 1 - Bel(Y_T)$ as usual and r_i is agent 3's rating on agent 1 (c) and agent 2 (s) respectively. Note that the model could be extended by adding separate ratings for both the positive and negative beliefs. We just show the simple model here and.

Figure 3 shows the belief and plausibility functions when agent 3's rating of both agents 1 and 2 are at 95%. Note that agent 3's belief in agent 1 and 2's reliability does not get any higher than 95%. In figure 4 and Tables 5 and 6, agent 1 begins to fail and, as its performance falls, agent 3's rating falls to 60%. This pushes agent 3's $Bel(\{Y_T\})$ down. At best agent 3 will believe that the system is in control less than 60% of the time. This addition to the processing complexity of the system comes at additional computational cost. However, there are definite benefits to having early warning capabilities to detect systemic failures. Our next section discusses some of these design tradeoffs.

Our last example is an extension of the previous example. It has three scenarios. The first is where agent 3 believes agent 1 is reporting reliably. This is the same scenario as our first example reported above. The second scenario is when agent 3 believes agent 1 is reporting good credit as bad credit. This situation is handled by discounting the belief of agent 1 by agent 3. That is, the revised belief mass considered by agent 3 coming from agent 1 would be given by: $m'^{+}_{c} = m_c^{+} + d*m_c^{-}$, $m'^{-}_{c} = m_c^{-} - d*m_c^{-}$, and $m'^{\Theta}_{c} = m_c^{\Theta}$ where d is a number between 0 and 1. d represents the discounting factor or the distrust factor. This scenario is reported in Table 7 and Table 8 and shown graphically in Figure 5. The third scenario represents agent 3 believing agent 1 is reporting bad credit as good credit. That is, the revised belief mass considered by agent 3 coming from agent 1 would be given by: $m'^{+}_{c} = m_c^{+} - d*m_c^{+}$, $m'^{-}_{c} = m_c^{-} + d*m_c^{-}$, and $m'^{\Theta}_{c} = m_c^{\Theta}$. The bad credit as good scenario is reported in Table 9 and Table 10 and shown graphically in

Figure 6. Because agent 3's distrust factor is increasing in this scenario, the belief never attains a value higher than .25 as is readily seen in Figure 6.

DISCUSSION

The proceeding formulation has both quality and service benefits [Nelson, *et al.*, 2000] as mentioned in the literature review. The first quality benefit is for the integrity of transactions and data. On the face of it, a system which allows for the aggregation of evidence about the operational effectiveness of a transaction processing system will help to reduce the integrity risk of that system. Additionally, the enhancement of enabling agent 3 to have a different belief about the reliability of the feeder agents or to keep a rating score on those agents reliability has an added risk reducing character, although at a cost. Once transactions are captured and processed with integrity, data storage integrity risk is also decreased since we have increased the probability that they were stored correctly in the first place. The other quality benefit of the model is the completeness of transactions and data. Combining evidence from multiple sources makes it straight forward to detect missing signals from upstream agents. This provides some ability to reduce the risk of lost transactions passing through the system. Agent 3 rating of the upstream agents also allows the system to penalize agents which are not processing transactions or data correctly and helps to reduce completeness risk.

The major service benefit of our model is in competitive intelligence. Designing and implementing a belief function internal control model builds intelligence metrics throughout the system which can be used to gauge the operational effectiveness and efficiency of the system at various levels of granularity. Knowledge about operations, about the present state of the company, is critical to any strategic moves the company considers. It leads to insights in process reengineering, risk reduction, and competitive strategy. So we believe that this design can have substantial benefits to the competitive intelligence of a company.

An additional major benefit of our formulation is in weighing the costs and benefits of internal control system designs. In the computational example which extended the original belief function model by adding a rating coefficient, the increase in the precision of agent 3's beliefs is easily measured and is the benefit of re-designing the system with that added capability. The cost of the new system is the additional components necessary to record and process the scores and ratings. This gives designers quantitative measures of both costs and benefits which can either be used as is or converted into financial measures. Models which allow this level of analysis with respect to design decisions in internal control environments are sorely lacking so this is a big additional benefit for our technique.

FITURE RESEARCH

The first major area of future research is in the realm of a functional operationalization of the model. Perhaps the best way to do this, from a practice vantage point, is to use virtualization to construct the different "companies" involved: retailer, ISPs with buying clients, credit bureau, etc. We have begun conceptual work on this project. A second area of extension is to add the agent 3 ratings for both sides of beliefs. For example, a rating for the agent 1 signal that credit is approved as well as one for the signal that credit is not approved. Related to this is a more complete formulation of the cost benefit measures in the designing of the systems of internal controls.

Another interesting problem which arises from these types of model building projects is that automation increasingly allows a unification of management controls with internal controls. Although the line is often blurred, management control is typically seen as being exercised through those systems which are directly designed to support organizational goal attainment and performance measurement systems. Internal controls are usually defined with respect to overall company effectiveness and efficiency, compliance, and transaction processing reliability. When both auditors and managers are concerned with the same system of internal controls and its evaluation, we experience a lack of clarity as to how management control, internal control, and audit control systems should be designed and operated. Modeling these systems, especially when simulations are included, will help us gain insights into this complex research area.

Another extension is to add clusters of feeder agents and additional hierarchical levels to the belief function tree. This has already been done in, for instance, [Mock, *et al.*, 2009] but the exploration of robust realistic systems in specific business contexts has not been explored. If the design considerations and cost benefit metrics are added to this level of analysis, a rich area for both practical and theoretical insights emerges. This is another area in which the authors are developing new projects.

A final area for additional research lies in compensating controls. This can be operationalized by adding to a downstream agent's ability to combine evidence, although there are other ways of conceiving solutions to this problem. If we use a downstream agent approach, then the model can be extended to allow the agent to use compensating evidence from other agents or collections of agents when it recalculates its beliefs. This has implications on the robustness of the design of the system of internal controls and so can also be extended into the cost benefit context for the design of systems of internal controls. Overall, our model provides good insights into the design and evaluation of systems of internal controls using belief functions.

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Table 1 - Manipulating Agent 1 and 2 Reliability

Agent 1 (input beliefs)											
m_c^+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
m_c^-	0	0	0	0	0	0	0	0	0	0	0
m_c^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
Agent 2 (input beliefs)											
m_S^+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
m_S^-	0	0	0	0	0	0	0	0	0	0	0
m_S^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Table 2 - Result of Agent 1 and 2 Manipulation on Agent 3

Agent 3 (Output beliefs)											
$m(Y_T)$	0	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	1
$m(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$m(N_T)$	0	0	0	0	0	0	0	0	0	0	0
$m(\{Y_T,D_T\})$	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
$m(\{Y_T,N_T\})$	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
$m(\{D_T,N_T\})$	0	0	0	0	0	0	0	0	0	0	0
$m(Q_T)$	1	0.81	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0
$Bel(Y_T)$	0	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	1
$Bel(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$Bel(N_T)$	0	0	0	0	0	0	0	0	0	0	0
$Pl(Y_T)$	1	1	1	1	1	1	1	1	1	1	1
$Pl(D_T)$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$Pl(\{D_T,N_T\})$	**1**	**0.99**	**0.96**	**0.91**	**0.84**	**0.75**	**0.64**	**0.51**	**0.36**	**0.19**	**0**

Table 3 - Agent 1 Reports Bad Credit as Good Credit

Agent 1 (input beliefs)											
mc+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
mc-	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
mc^Q	0	0	0	0	0	0	0	0	0	0	0
Agent 2 (input beliefs)											
m_S+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
m_S-	0	0	0	0	0	0	0	0	0	0	0
m_S^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Table 4 - Results of Reporting bad Credit as Good Credit

Agent 3 (Output beliefs)											
$m(Y_T)$	0	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	1
$m(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$m(N_T)$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$m(\{Y_T,D_T\})$	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
$m(\{Y_T,N_T\})$	0	0	0	0	0	0	0	0	0	0	0
$m(\{D_T,N_T\})$	0	0	0	0	0	0	0	0	0	0	0
$m(Q_T)$	0	0	0	0	0	0	0	0	0	0	0
$Bel(Y_T)$	0	0.01	0.04	0.09	0.16	0.25	0.36	0.49	0.64	0.81	1
$Bel(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$Bel(N_T)$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$Pl(Y_T)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$Pl(D_T)$	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
$Pl(\{D_T,N_T\})$	**1**	**0.99**	**0.96**	**0.91**	**0.84**	**0.75**	**0.64**	**0.51**	**0.36**	**0.19**	**0**

Table 5 - Credit Reporting Becomes Unreliable

Agent 1 (input beliefs)												Percent Correct
m_c^+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.6
m_c^-	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0	0.6
m_c^Q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0	
Agent 2 (input beliefs)												
m_S^+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	0.95
m_S^-	0	0	0	0	0	0	0	0	0	0	0	0.95
m_S^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0	

Table 6 - Agent 3's Reaction to Unreliable Credit Reporting

Agent 3 (Output beliefs)											
$m(Y_T)$	0	0.0057	0.0228	0.0513	0.0912	0.1425	0.2052	0.2793	0.3648	0.4617	0.57
$m(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$m(N_T)$	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.03	0
$m(\{Y_T,D_T\})$	0	0.054	0.096	0.126	0.144	0.15	0.144	0.126	0.096	0.054	0
$m(\{Y_T,N_T\})$	0	0.076	0.133	0.171	0.19	0.19	0.171	0.133	0.076	0.04275	0
$m(\{D_T,N_T\})$	0	0	0	0	0	0	0	0	0	0	0
$m(Q_T)$	0.9	0.72	0.56	0.42	0.3	0.2	0.12	0.06	0.02	0.005	0
$Bel(Y_T)$	0	0.0057	0.0228	0.0513	0.0912	0.1425	0.2052	0.2793	0.3648	0.4617	0.57
$Bel(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$Bel(N_T)$	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.03	0
$Pl(Y_T)$	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.97	1
$Pl(D_T)$	0.94	0.8507	0.7614	0.6721	0.5828	0.4935	0.4042	0.3149	0.2256	0.14065	0.05
$Pl(\{D_T,N_T\})$	**1**	**0.9943**	**0.9772**	**0.9487**	**0.9088**	**0.8575**	**0.7948**	**0.7207**	**0.6352**	**0.5383**	**0.43**

Table 7 - Discounting of Agent 3's Belief in Agent 1, Good as Bad Scenario

Agent 1 (input beliefs)											
Initial input as provided (assessed) by Agent 1											
m_c^+	0	0	0	0	0	0	0	0	0	0	0
m_c^-	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
m_c^Q	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Discounted belief masses for Agent 1 by Agent 3, i.e., adjusted belief masses											
d =	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$m'_c{}^+ = m_c^+ + d*m_c^+$	0	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9
$m'_c{}^- = m_c^- - d*m_c^-$	0.9	0.81	0.72	0.63	0.54	0.45	0.36	0.27	0.18	0.09	0
$m'_c{}^Q = m_c^Q$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Agent 2 (input beliefs)											
m_S+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
m_S-	0	0	0	0	0	0	0	0	0	0	0
m_S^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Table 8 - Agent 3's Reaction to Discounting, Good as Bad Scenario

Agent 3 (Output beliefs)											
m(Y_T)	0	0.009	0.036	0.081	0.144	0.225	0.324	0.441	0.576	0.729	0.9
m(D_T)	0	0	0	0	0	0	0	0	0	0	0
m(N_T)	0.9	0.81	0.72	0.63	0.54	0.45	0.36	0.27	0.18	0.09	0
m({Y_T,D_T})	0	0.081	0.144	0.189	0.216	0.225	0.216	0.189	0.144	0.081	0
m({Y_T,N_T})	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
m({D_T,N_T})	0	0	0	0	0	0	0	0	0	0	0
m(Q_T)	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0
Bel(Y_T)	0	0.009	0.036	0.081	0.144	0.225	0.324	0.441	0.576	0.729	0.9
Bel(D_T)	0	0	0	0	0	0	0	0	0	0	0
Bel(N_T)	0.9	0.81	0.72	0.63	0.54	0.45	0.36	0.27	0.18	0.09	0
Pl(Y_T)	0.1	0.19	0.28	0.37	0.46	0.55	0.64	0.73	0.82	0.91	1
Pl(D_T)	0.1	0.171	0.224	0.259	0.276	0.275	0.256	0.219	0.164	0.091	0
Pl({D_T,N_T})	**1**	**0.991**	**0.964**	**0.919**	**0.856**	**0.775**	**0.676**	**0.559**	**0.424**	**0.271**	**0.1**

Table 9 - Discounting of Agent 3's Belief in Agent 1, Bad as Good Scenario

Agent 1 (input beliefs)											
Initial input as provided (assessed) by Agent 1											
m_c^+	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
m_c^-	0	0	0	0	0	0	0	0	0	0	0
m_c^Q	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Discounted belief masses for Agent 1 by Agent 3, i.e., adjusted belief masses											
d =	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$m'_c{}^+ = m_c^+ + d*m_c^+$	0.9	0.81	0.72	0.63	0.54	0.45	0.36	0.27	0.18	0.09	0
$m'_c{}^- = m_c^- - d*m_c^-$	0	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9
$m'_c{}^Q = m_c^Q$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Agent 2 (input beliefs)											
m_S+	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
m_S-	0	0	0	0	0	0	0	0	0	0	0
m_S^Q	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Table 10 - Agent 3's Reaction to Discounting, Bad as Good Scenario

Agent 3 (Output beliefs)											
$m(Y_T)$	0	0.081	0.144	0.189	0.216	0.225	0.216	0.189	0.144	0.081	0
$m(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$m(N_T)$	0	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9
$m(\{Y_T,D_T\})$	0.9	0.729	0.576	0.441	0.324	0.225	0.144	0.081	0.036	0.009	0
$m(\{Y_T,N_T\})$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
$m(\{D_T,N_T\})$	0	0	0	0	0	0	0	0	0	0	0
$m(Q_T)$	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0
$Bel(Y_T)$	0	0.081	0.144	0.189	0.216	0.225	0.216	0.189	0.144	0.081	0
$Bel(D_T)$	0	0	0	0	0	0	0	0	0	0	0
$Bel(N_T)$	0	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.9
$Pl(Y_T)$	1	0.91	0.82	0.73	0.64	0.55	0.46	0.37	0.28	0.19	0.1
$Pl(D_T)$	1	0.819	0.656	0.511	0.384	0.275	0.184	0.111	0.056	0.019	0
$Pl(\{D_T,N_T\})$	**1**	**0.919**	**0.856**	**0.811**	**0.784**	**0.775**	**0.784**	**0.811**	**0.856**	**0.919**	**1**

Figure 1 - Agents Communicate without Noise

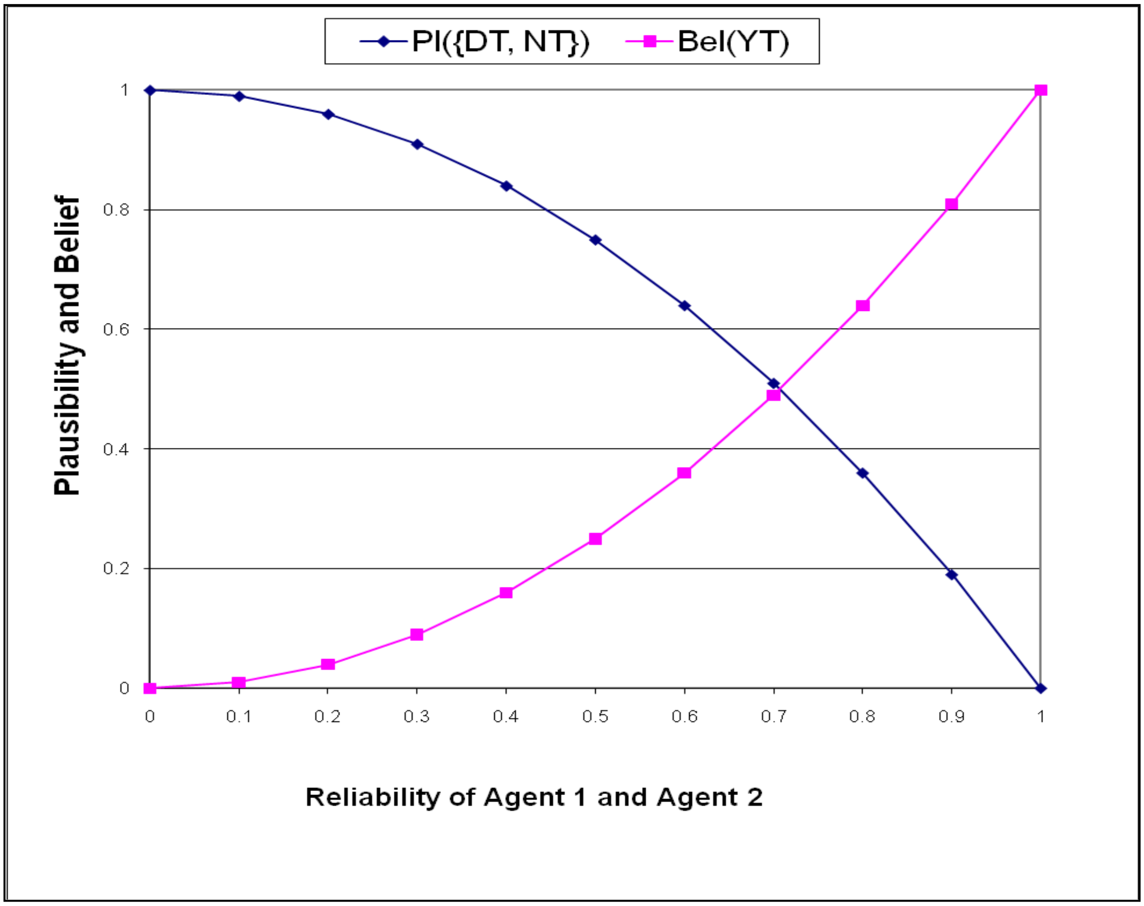

Figure 2 - Agents Communicate Bad Credit as Good Credit

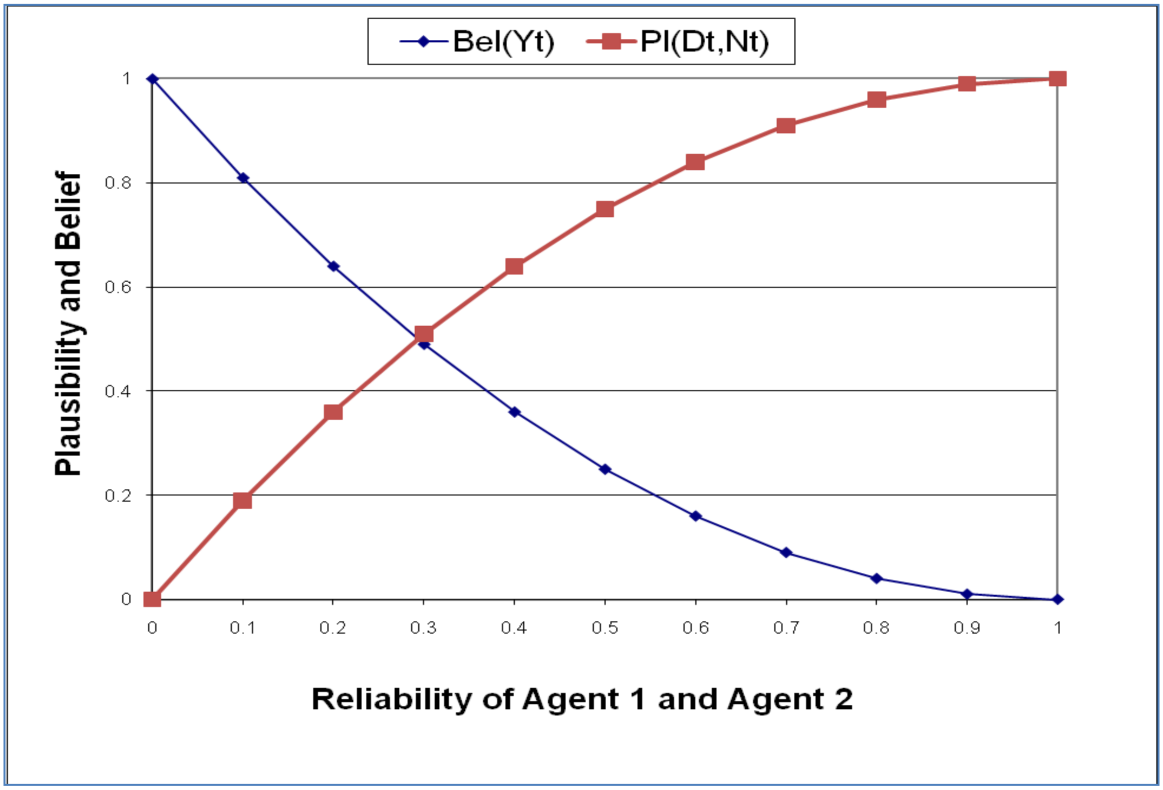

Figure 3 - Agent 1 and 2 at 95% Confidence

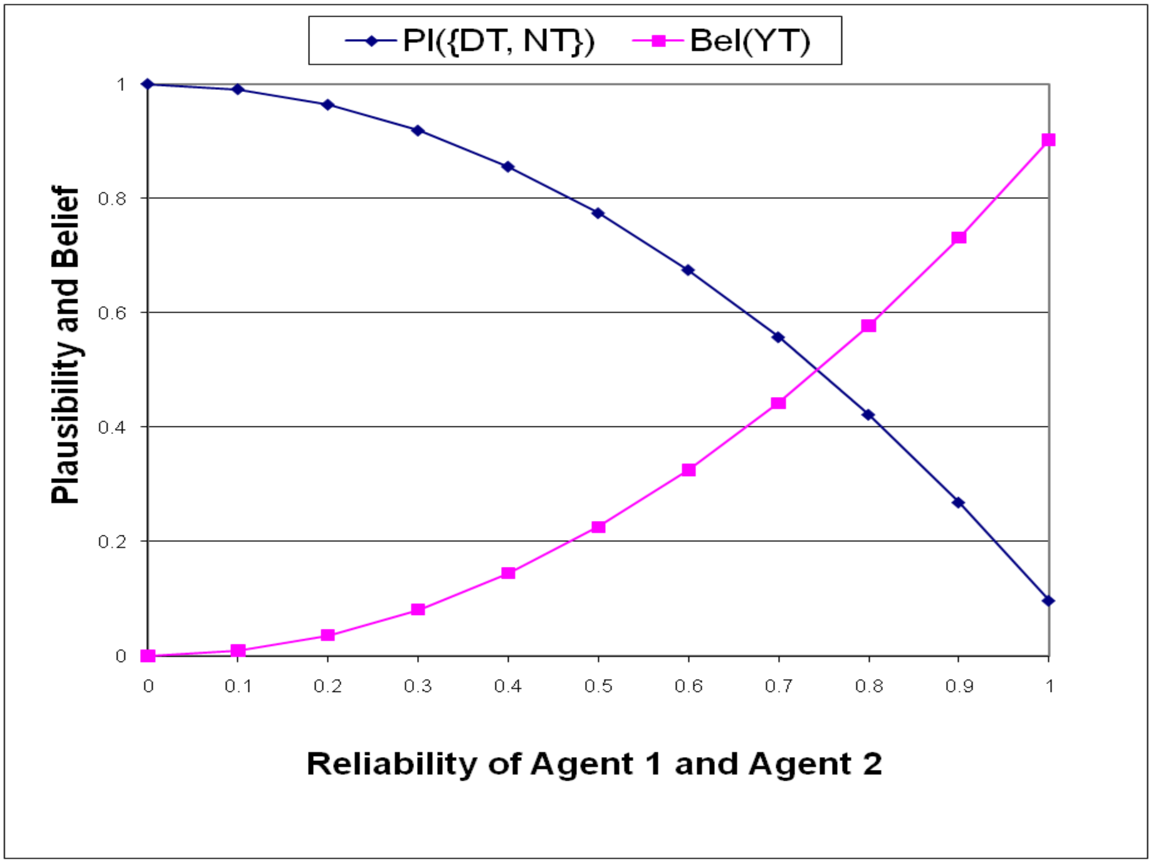

Figure 4 - Agent 1 Begins to Fail and Falls to 60% Confidence

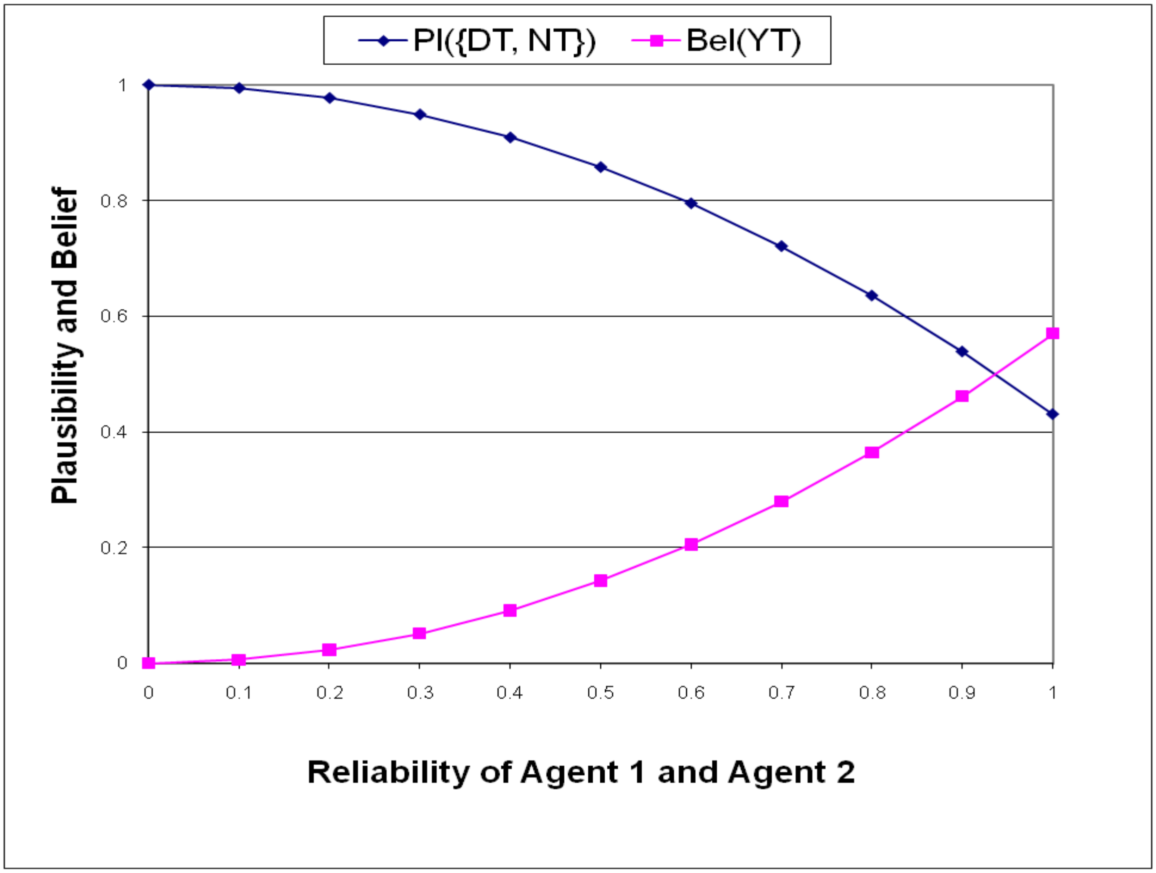

Figure 5 - Agent 3 Believes Good Credit is Reported as Bad by Agent 1

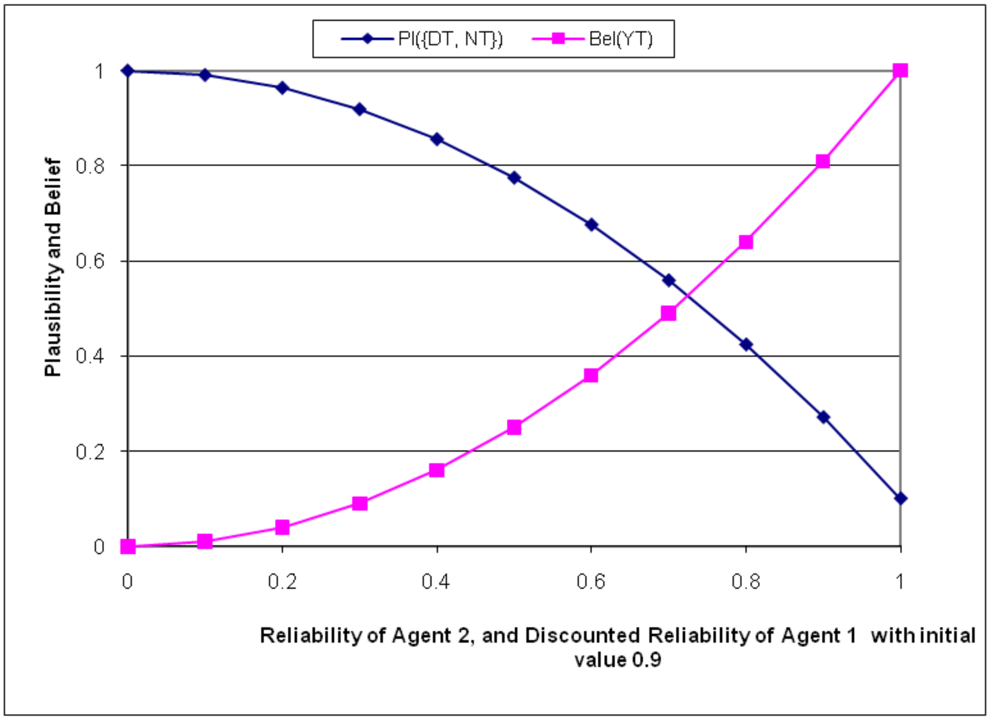

Figure 6 - Agent 3 Believes Bad Credit is Reported as Good by Agent 1